PROGRESSIVE FAILURE ANALYSIS OF $[0/\pm 60]$ LAMINATES UNDER BI-AXIAL STRESS BY GENERALIZED YEH-STRATTON CRITERION

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Committee Members:

Jalal Torabzadeh, Ph.D. (Chair)
Chang Kim, Ph.D.
Hsin-Piao Chen, Ph.D.

College Designee:

Antonella Sciortino, Ph.D

By Jagadesh Rao Thalur

B.E., 2011, Anna University, Chennai, India

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ABSTRACT

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By

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The light weight of composite materials has attracted interests to improve fuel economy of aircrafts and to extend flight range. The usage of composite materials is increasing in airframes and other parts of aerospace industry. Although most tests on composites are conducted uniaxially, they are subjected to multi-axial loads in real life applications. Hence, there is a need to better understand the complex failure mechanisms in composite structures. More reliable failure theories and damage progression models should be devised. Also, reliable criteria for predicting failure of fiber composite laminates are necessary for rational analysis and design. In this thesis, the behavior of a symmetric composite material under bi-axial loading is studied and the failure of the composite material is predicted by Yeh-Stratton criterion. A MATLAB program is prepared for the study of failure in tubular specimens composed of AS4/3501-6 carbon/epoxy laminates, which were subjected to internal pressure and axial force simultaneously to vary the states of stress. It is shown that the Yeh-Stratton criterion is in a good agreement with the experimental results. Future work may include collection of more accurate and different kind of experimental data on composite materials and modification of the interaction factor B_{12} value to evaluate its effect on the theoretical prediction by the Yeh-Stratton criterion.

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CHAPTER 1

INTRODUCTION

1.1 Background

To develop high speed air-vehicles and high speed robotics, structural components need to possess features like light weight, high stiffness, and high strength. To satisfy both light weight and high stiffness-to-strength ratio, a light weight composite material which consists of fibrous material is one of the candidates for application. In this thesis, the mechanical properties of the composite material will be analyzed from the view point of macro-mechanical study and progressive failure analysis using experimental results from literature.

Advanced fiber composites have excellent strength and stiffness to weight properties and are often used in strength-critical applications. The prediction of failure in fiber composites involves a number of aspects, and a number of potential failure modes must be considered. At present, a complete theoretically based failure prediction criterion has not been developed.

The optimal design and analysis of composite materials are always challenging due to the complex interfacial effects of matrix and fiber. An understanding of the response and progressive failure of composite materials is necessary for the design of a safe structure. A variety of analyses have been developed to successfully predict various effective failure properties based on classical lamination theory or finite element methods. In the last two decades, considerable progress has been made in understanding the initiation and evolution of various damage modes for a laminated composite consisting of stacks of unidirectional laminae. The commonly used failure criteria are the maximum stress criterion and Tsai-Wu criterion.

1.2 Previous Work

A new generation of physically based failure criteria was founded by Hashin and Rotem [1,2], where different failure modes are described by separate equations. In their 1973 paper, two separate criteria for fiber and matrix failure were introduced and a quadratic interaction between the tractions acting on the plane of failure assumed. To overcome the difficulty of finding the plane of matrix fracture, a quadratic interaction between stress invariants was used [3], based on logical reasoning rather than on micromechanics. In 1998, Puck and Shurmann [4] improved the matrix failure criterion incorporating the beneficial effect of transverse compression on matrix shear strength.

In 1965, Gol'denblat and Kopnov [5] developed a general polynomial stress-based criterion. Various researchers have used and adapted it to specific materials and engineering applications. In this way, Tsai and Wu developed one of the popular criteria for multilayered composites.

Hill [6] developed a failure criterion for anisotropic materials that was used by Azzi and Tsai [7] to analyze initial failure in thin laminates with transversely isotropic properties. This criterion is better known as Tsai-Hill Criterion.

Kim and Yeh [8,9,10] developed a criterion for isotropic materials in order to develop a failure theory suitable for both ductile and brittle materials, which is known as Yeh-Stratton (Y-S) criterion. The generalized Y-S criterion is applicable for composite materials. In this criterion, failure is influenced mainly by the normal stresses, rather than by the interaction or their shear stresses.

1.3 Present Strategies in Progressive Failure Analysis of Composite Materials

There are various published failure theories in the literature [11-14]. They are used not only for predicting the initiation of failure but also to predict ultimate failure load through progressive failure analysis. The popularity of some failure theories over others appears to be due to their ease of use. The earlier theories such as the Maximum Stress, the Maximum Strain, Hashin, the Tsai-Hill and the Tsai-Wu failure theories are still widely used despite their shortcomings because of their simplicity [13,14]. Theories that allow interaction between stress components such as the Tsai-Wu criterion generally perform better [20,21].

The strategy used in this paper is to change the extensional stiffness (ABD) matrix, which essentially defines elastic properties of entire laminate, after the first ply have failed, and adding more load to the degraded laminate until all the plies have failed at least in one of the loading directions. Appendix B shows a flowchart, which is made to clarify the strategy used for progressive failure analysis. A sample hand calculation is shown to check the validity of the computation.

1.4 Objectives

This thesis has three major objectives:

1. To create a program using MATLAB which can give results of progressive failure of any composite material.
2. To compare the experimental data of progressive failure of composite material with the theoretical prediction using Classical lamination theory and various theories of failure.
3. To compare a number of failure criterion namely, Tsai-Wu criterion, Hashin criterion, Yeh-Stratton criterion and Maximum stress criterion and to check the performance of those criteria.

CHAPTER 2

CLASSICAL LAMINATION THEORY

2.1 Introduction to Classical Lamination Theory

2.1.1 Description of laminate

A laminate is an organized stack of uni-directional or multi-directional composite plies. In Appendix A, Figure 1 shows composite laminates and the Lay-Up nomenclature. Examples of lay up sequences is shown in Figure 1 in Appendix A. The “t” stands for “truncate,” the “s” for “symmetrical” (implying the listed sequence should be mirrored across the laminate’s midplane) and the “2” outside of the parenthesis means that sequence is repeated twice. The fiber angles are measured from a general global co-ordinate system.

Material Properties

In addition to the stacking sequence of the laminate, the following material properties of the composite material must be defined.

- Mechanical Elasticity (E11, E22, G12 and ν12)
- Environmental Elasticity (α11, α22, β11, β12) which represent thermal and moisture expansion, respectively.

Mechanical and Environmental Loads

Finally, the mechanical and environmental loads must be defined:

- Normal Forces(Nxx, Nyy, Nxy)
- Bending Moments (Mxx, Myy, Mxy)
- Environmental Forces caused by Environmental Strains (N_x^T, N_y^T, N_{xy}^T, M_x^T, M_y^T, M_{xy}^T)

2.1.2 CLT Calculations – the ABD Matrix

The ABD Matrix is a 6x6 matrix that serves as a connection between the applied loads and the associated strains in the laminate. It essentially defines the elastic properties of the entire laminate. To assemble the ABD Matrix, the following steps need to be performed:

1. Calculate the reduced stiffness matrix Q_{ij} for each material used in the laminate (If a laminate uses only one type of composite material, there will be only 1 stiffness matrix). The stiffness matrix describes the elastic behavior of the ply in plane loading.

   Where

$$Q_{ij} = \begin{pmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{pmatrix} \tag{1}$$

$$Q_{11} = \frac{E_{11}^2}{(E_{11} - \nu_{12*}E_{22})} \tag{2}$$

$$Q_{12} = \frac{\nu_{12*}E_{11*}E_{12}}{(E_{11} - \nu_{12*}^2E_{22})} \tag{3}$$

$$Q_{22} = \frac{E_{11*}E_{12}}{(E_{11} - \nu_{12*}^2E_{22})} \tag{4}$$

$$Q_{66} = G_{12} \tag{5}$$

2. Calculate the transformed reduced stiffness matrix $\overline{Q_{\iota\jmath}}$ for each ply based on the reduced stiffness matrix and fiber angle.
   Where

$$\overline{Q_{11}} = Q_{11}\cos(\theta)^4 + 2(Q_{12} + 2Q_{66})\cos(\theta)^2 * \sin(\theta)^2 + Q_{22}\sin(\theta)^4 \tag{6}$$

$$\overline{Q_{12}} = \overline{Q_{21}} = Q_{12}\left(\cos(\theta)^4 + \sin(\theta)^4\right) + (Q_{11} + Q_{22} - 4Q_{66})\cos(\theta)^2 * \sin(\theta)^2 \tag{7}$$

$$\overline{Q_{16}} = \overline{Q_{61}} = (Q_{11} - Q_{12} - 2Q_{66})\cos(\theta)^3 * \sin(\theta) - (Q_{11} - Q_{12} - 2Q_{66})\cos(\theta) * \sin(\theta)^3 \tag{8}$$

$$\overline{Q_{22}} = Q_{11}\sin(\theta)^4 + 2(Q_{12} + 2Q_{66})\cos(\theta)^2 * \sin(\theta)^2 + Q_{22}\cos(\theta)^4 \tag{9}$$

$$\overline{Q_{26}} = \overline{Q_{62}} = (Q_{11} - Q_{12} - 2Q_{66})\cos(\theta) * \sin(\theta)^3 - (Q_{11} - Q_{12} - 2Q_{66})\cos(\theta)^3 * \sin(\theta) \tag{10}$$

$$\overline{Q_{66}} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\cos(\theta)^2 * \sin(\theta)^2 + Q_{66}(\cos(\theta)^4 * \sin(\theta)^4) \tag{11}$$

$$\overline{Q_{ij}} = \begin{pmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}} \\ \overline{Q_{21}} & \overline{Q_{22}} & \overline{Q_{26}} \\ \overline{Q_{61}} & \overline{Q_{62}} & \overline{Q_{66}} \end{pmatrix} \tag{12}$$

3. Calculate the A_{ij}, B_{ij}, D_{ij} matrices using the following equations where z represents the vertical position of the ply from the midplane:

$$A_{ij} = \sum_{k=1}^{n} \{\overline{Q_{ij}^{n}}\}(Z_k - Z_{k-1}) \tag{13}$$

$$B_{ij} = \frac{1}{2}\sum_{k=1}^{n} \{\overline{Q_{ij}^{n}}\}(Z_k^2 - Z_{k-1}^2) \tag{14}$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{n} \{\overline{Q_{ij}^{n}}\}(Z_k^3 - Z_{k-1}^3) \tag{15}$$

4. Assemble ABD:

$$ABD = \begin{pmatrix} A & B \\ B & D \end{pmatrix} \tag{16}$$

5. Calculate inverse of ABD: $abd = ABD^{-1}$ (17)

6. Calculate thermal and moisture expansion coefficients for each ply:

$$\begin{aligned} \alpha_{xx} &= \alpha_{11}\cos(\theta)^2 + \alpha_{22}\sin(\theta)^2 \\ \alpha_{yy} &= \alpha_{11}\sin(\theta)^2 + \alpha_{22}\cos(\theta)^2 \\ \alpha_{xy} &= 2\cos(\theta)\sin(\theta)\,(\alpha_{11} - \alpha_{22}) \end{aligned} \tag{18}$$

$$\begin{aligned} \beta_{xx} &= \beta_{11}\cos(\theta)^2 + \beta_{22}\sin(\theta)^2 \\ \beta_{yy} &= \beta_{11}\sin(\theta)^2 + \beta_{22}\cos(\theta)^2 \\ \beta_{xy} &= 2\cos(\theta)\sin(\theta)\,(\beta_{11} - \beta_{22}) \end{aligned} \tag{19}$$

7. Calculate thermal and moisture stress and moment resultants:

Thermal Resultants:

$$\left.\begin{aligned} N_{xx}^{T} &= \Delta T \sum_{k=1}^{n} \{[\overline{Q_{11}}\alpha_{xx} + \overline{Q_{12}}\alpha_{yy} + \overline{Q_{16}}\alpha_{xy}]k[Z_k - Z_{k-1}]\} \\ N_{yy}^{T} &= \Delta T \sum_{k=1}^{n} \{[\overline{Q_{12}}\alpha_{xx} + \overline{Q_{22}}\alpha_{yy} + \overline{Q_{26}}\alpha_{xy}]k[Z_k - Z_{k-1}]\} \\ N_{xy}^{T} &= \Delta T \sum_{k=1}^{n} \{[\overline{Q_{16}}\alpha_{xx} + \overline{Q_{26}}\alpha_{yy} + \overline{Q_{66}}\alpha_{xy}]k[Z_k - Z_{k-1}]\} \end{aligned}\right\} \tag{20}$$

$$\left.\begin{aligned} M_{xx}^{T} &= \frac{\Delta T}{2} \sum_{k=1}^{n} \{[\overline{Q_{11}}\alpha_{xx} + \overline{Q_{12}}\alpha_{yy} + \overline{Q_{16}}\alpha_{xy}]k[Z_k - Z_{k-1}]\} \\ M_{yy}^{T} &= \frac{\Delta T}{2} \sum_{k=1}^{n} \{[\overline{Q_{12}}\alpha_{xx} + \overline{Q_{22}}\alpha_{yy} + \overline{Q_{26}}\alpha_{xy}]k[Z_k - Z_{k-1}]\} \\ M_{xy}^{T} &= \frac{\Delta T}{2} \sum_{k=1}^{n} \{[\overline{Q_{16}}\alpha_{xx} + \overline{Q_{26}}\alpha_{yy} + \overline{Q_{66}}\alpha_{xy}]k[Z_k - Z_{k-1}]\} \end{aligned}\right\} \tag{21}$$

Moisture Resultants:

$$\left.\begin{aligned}
N_{xx}^{M} &= \Delta T \sum_{k=1}^{n} \{[\overline{Q_{11}}\beta_{xx} + \overline{Q_{12}}\beta_{yy} + \overline{Q_{16}}\beta_{xy}]k[Z_k - Z_{k-1}]\} \\
N_{yy}^{M} &= \Delta T \sum_{k=1}^{n} \{[\overline{Q_{12}}\beta_{xx} + \overline{Q_{22}}\beta_{yy} + \overline{Q_{26}}\beta_{xy}]k[Z_k - Z_{k-1}]\} \\
N_{xy}^{M} &= \Delta T \sum_{k=1}^{n} \{[\overline{Q_{16}}\beta_{xx} + \overline{Q_{26}}\beta_{yy} + \overline{Q_{66}}\beta_{xy}]k[Z_k - Z_{k-1}]\}
\end{aligned}\right\} \quad (22)$$

$$\left.\begin{aligned}
M_{xx}^{M} &= \frac{\Delta T}{2} \sum_{k=1}^{n} \{[\overline{Q_{11}}\beta_{xx} + \overline{Q_{12}}\beta_{yy} + \overline{Q_{16}}\beta_{xy}]k[Z_k - Z_{k-1}]\} \\
M_{yy}^{M} &= \frac{\Delta T}{2} \sum_{k=1}^{n} \{[\overline{Q_{12}}\beta_{xx} + \overline{Q_{22}}\beta_{yy} + \overline{Q_{26}}\beta_{xy}]k[Z_k - Z_{k-1}]\} \\
M_{xy}^{M} &= \frac{\Delta T}{2} \sum_{k=1}^{n} \{[\overline{Q_{16}}\beta_{xx} + \overline{Q_{26}}\beta_{yy} + \overline{Q_{66}}\beta_{xy}]k[Z_k - Z_{k-1}]\}
\end{aligned}\right\} \quad (23)$$

8. Calculate midplane strains and curvatures induced in the laminate. These represent the deflections of the laminate.

$$\begin{pmatrix} \varepsilon_{xx}^{0} \\ \varepsilon_{yy}^{0} \\ \gamma_{xy}^{0} \\ k_{xx} \\ k_{yy} \\ k_{xy} \end{pmatrix} = [abd] \begin{pmatrix} N_{xx} + N_{xx}^{T} + N_{xx}^{M} \\ N_{yy} + N_{yy}^{T} + N_{yy}^{M} \\ N_{xy} + N_{xy}^{T} + N_{xy}^{M} \\ M_{xx} + M_{xx}^{T} + M_{xx}^{M} \\ M_{yy} + M_{yy}^{T} + M_{yy}^{M} \\ M_{xy} + M_{xy}^{T} + M_{xy}^{M} \end{pmatrix} \quad (24)$$

9. For each ply, Calculate ply strains in the global co-ordinate system

$$\begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \end{pmatrix} + Z \begin{pmatrix} k_{xx} \\ k_{yy} \\ k_{xy} \end{pmatrix} \tag{25}$$

10. Calculate ply stresses in the global co-ordinate system

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{pmatrix} = \begin{pmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}} \\ \overline{Q_{21}} & \overline{Q_{22}} & \overline{Q_{26}} \\ \overline{Q_{61}} & \overline{Q_{62}} & \overline{Q_{66}} \end{pmatrix} * \begin{pmatrix} \varepsilon_{xx} - \Delta T\alpha_{xx} - \Delta M\beta_{xx} \\ \varepsilon_{yy} - \Delta T\alpha_{yy} - \Delta M\beta_{yy} \\ \gamma_{xy} - 2\Delta T\alpha_{xy} - 2\Delta M\beta_{xy} \end{pmatrix} \tag{26}$$

CHAPTER 3

EXPERIMENTAL DETAILS

The experimental data were obtained from available literature[15]. A thin walled cylinder with reinforced ends is used as specimen for bi-axial tests. The specimen is loaded with internal pressure and axial load [15].

The material used for the bi-axial tests is AS4/3501-6 carbon/epoxy prepreg [15] in a [0/60/-60/0/-60/60/0] layup.

A schematic of the experimental setup is shown in Figure 2 in Appendix A

In the lay-up sequence, 0^o corresponds to the hoop direction, which is the direction of the N_x load in the cylindrical configuration [15]. The bi-axial load ratio is defined to be $k1=N_y/N_x$, and $N_y = k1*N_x$. So, N_y changes depending on N_x. Calculation technique needs N_x to be the higher value to start k1 from zero. Note that the laminate is not quasi-isotropic. The specimens were made by Hercules Aerospace, Magna, Utah, USA [15]. In Appendix A, Table 1 gives the fiber lot acceptance tensile test values for the fiber lot used [15].

The tests were all performed with proportional loading of the internal pressure and axial load. This mode is obtained by using load feedback in a servo-controlled test machine with the pressure signal as the load command [15]. Very precise control of the ratio of stresses can be obtained in this manner. Strains were measured with strain gauge rosettes placed on the specimen axial mid-plane and located around the circumference [15].

The laminate failure stresses are measured and listed in Figure 4 in Appendix A and the failure stresses have been plotted in Table 2 in Appendix C

CHAPTER 4

THEORIES OF FAILURE USED TO PREDICT FAILURE OF THE COMPOSITE MATERIAL

The plies are considered ‘building blocks’ for laminates [26], It is thus desirable to base considerations of laminate failure on the properties of the plies. Since the strength of matrix and fiber can differ by a factor of 50 [22, 26], it is thus necessary to distinguish fiber and matrix failure modes. Hence it appears that it is desirable to be able to predict both when a ply will fail and how will it fail. We shall assume that fiber failure mode represent ultimate failure of the lamina. A number of failure criteria have been compared with the present data. The criteria that will be displayed are the Tsai-Wu criterion [23], Hashin stress quadratic polynomial [22], Yeh-Stratton criterion [24] and Maximum Stress Criterion.

Expressions of those criteria are given below, as:

Tsai-Wu Criterion [23]:

$$f_1\sigma_1 + f_{11}\sigma_1^2 + f_2\sigma_2 + f_{22}\sigma_2^2 + f_{66}\tau_{12}^2 + 2f_{12}\sigma_1\sigma_2 = 1 \quad (27)$$

Sometimes, the Tsai-Wu criterion is used without the interaction term, f_{12} [15].

In this paper, the Tsai-Wu interaction term is not included in the analysis.

Hashin Criterion [22]:

$$f_1\sigma_1 + f_2\sigma_2 + f_{66}\tau_{12}^2 = 1 \quad (28)$$

Where,

$$f_1 = \frac{1}{F_{1t}} - \frac{1}{F_{1c}}$$

$$f_2 = \frac{1}{F_{2t}} - \frac{1}{F_{2c}}$$

$$f_{11} = \frac{1}{F_{1t} * F_{1c}}$$

$$f_{22} = \frac{1}{F_{2t} * F_{2c}}$$

$$f_{66} = \frac{1}{F_6^2}$$

$$f_{12} = -0.5(f_{11} * f_{22})^{1/2} = -0.5\left(\frac{1}{F_{1t} * F_{1c} * F_{2t} * F_{2c}}\right)^{1/2}$$

Yeh-Stratton Criterion [24]:

$$\frac{\sigma_1}{F_1} + \frac{\sigma_2}{F_2} + B_{12}\sigma_1\sigma_2 + \frac{\tau_{12}^2}{F_6^2} = 1 \qquad (29)$$

B_{12} value, the interaction component of the generalized Yeh-Stratton criterion shall be generally calibrated by doing iterative changes and by comparing with the experimental data. For the laminate used in this paper, B_{12} has been chosen as -1.225e-17 MPa^{-2} which is approximately equal to $\frac{1}{F_{1t}*F_{2t}}$. If B_{12} value is increased more than $\frac{1}{F_{1t}*F_{2t}}$, the results will be complex numbers. The effect of B_{12} of the Y-S criterion can be referenced to [10].

Maximum Stress Criterion:

$$\left.\begin{aligned} \sigma_1 &\geq F_1 \\ \sigma_2 &\geq F_2 \\ \tau_{12} &\geq F_6 \end{aligned}\right\} \qquad (30)$$

The failure occurs if one the above inequalities is satisfied. In other words, the failure occurs if the stresses in the natural axes (longitudinal or transverse) exceeds the corresponding allowable stress.

F_1 and F_2 are chosen based on whether the corresponding stress value is positive (tensile stress) or negative (compressive stress). i.e. if σ_1 is positive, F_1 will be F_{1t} and if σ_1 is negative, F_1 will be F_{1c}.

CHAPTER 5

MATLAB PROGRAM

A group of MATLAB algorithms has been made to run the progressive failure analysis of any composite laminate using different types of failure criteria. The program will return first ply failure load, ultimate failure load and kind of failure the laminate went through step by step for various bi-axial load ratio (i.e., (N_y/N_x)). In Appendix B a flowchart is made to show the algorithm steps.

CHAPTER 6

HAND CALCULATION USING Y-S CRITERION

The geometrical and material properties of the laminate is shown in Figure 6 and 7 in Appendix A.

The lamina reduced stiffness matrix is:

$$Q_{ij} = \begin{pmatrix} 1.2795 & 0.0327 & 0 \\ 0.0327 & 0.1128 & 0 \\ 0 & 0 & 0.0579 \end{pmatrix} * 10^{11} \tag{31}$$

The reduced stiffness matrix in the global direction is calculated from Eqns 6-12:

For the 0° ply

$$\overline{Q_{ij}}^{(1)} = \begin{pmatrix} 1.2795 & 0.0327 & 0 \\ 0.0327 & 0.1128 & 0 \\ 0 & 0 & 0.0579 \end{pmatrix} * 10^{11} \tag{32}$$

For the 60° ply

$$\overline{Q_{ij}}^{(2)} = \begin{pmatrix} 1.9913 & 2.3809 & 1.3402 \\ 2.3809 & 7.8246 & 3.7116 \\ 1.3402 & 3.7116 & 2.6327 \end{pmatrix} * 10^{10} \tag{33}$$

The ABD matrix is calculated from the Eqns 13,14,15,17

$$ABD = \begin{pmatrix} 1.2375 & 0.2805 & 0 & 0 & 0 & 0 \\ 0.2805 & 0.9261 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.3275 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4836 & 0.0582 & 0.0153 \\ 0 & 0 & 0 & 0.0582 & 0.1931 & 0.0424 \\ 0 & 0 & 0 & 0.0153 & 0.0424 & 0.0719 \end{pmatrix} * 10^{11} \tag{34}$$

$$abd = \begin{pmatrix} 0.0087 & -0.0026 & 0 & 0 & 0 & 0 \\ -0.0026 & 0.0116 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0305 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0215 & -0.0063 & -0.0009 \\ 0 & 0 & 0 & -0.0063 & 0.0613 & -0.0348 \\ 0 & 0 & 0 & -0.0009 & -0.0348 & 0.1598 \end{pmatrix} * 10^{-9} \quad (35)$$

And, the apparent coefficients of thermal expansion are calculated from Eqns 18,19

$$\begin{pmatrix} \alpha_x^{(1)} \\ \alpha_y^{(1)} \\ \alpha_{xy}^{(1)} \end{pmatrix} = \begin{pmatrix} 0.0350 \\ 0.1140 \\ 0 \end{pmatrix} * 10^{-4} \qquad (36)$$

$$\begin{pmatrix} \alpha_x^{(2)} \\ \alpha_y^{(2)} \\ \alpha_{xy}^{(2)} \end{pmatrix} = \begin{pmatrix} 0.9245 \\ 0.5475 \\ -0.3421 \end{pmatrix} * 10^{-5} \qquad (37)$$

The thermal forces are calculated using Eqns 20,21

$$\left.\begin{aligned} N_{xx}^T &= 3.3725 * 10^5 * t * \Delta T \\ N_{yy}^T &= 2.8796 * 10^5 * t * \Delta T \\ N_{xy}^T &= 0 \\ M_{xx}^T &= 0 \\ M_{yy}^T &= 0 \\ M_{xy}^T &= 0 \end{aligned}\right\} \qquad (38)$$

Since the laminate is undergoing a bi-axial load, the loads in two axes should be applied in a ratio to each other. For this sample calculation the ratio is 0.4, i.e..

$$k1 = \frac{N_y}{N_x} = 0.4$$

To find the stresses due to mechanical load alone, thermal strains should be subtracted from the total strain.

$$\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\end{pmatrix} = \overline{Q_{\iota J}}\left[\begin{pmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{pmatrix} - \begin{pmatrix}\alpha_x\\ \alpha_y\\ \alpha_{xy}\end{pmatrix}\Delta T\right] \tag{39}$$

From Equations 24,25 and 39, we have

$$\left.\begin{aligned}\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\end{pmatrix} = \overline{Q_{\iota J}}\Bigg\{&\left[\{ab\}\begin{pmatrix}N\\ M\end{pmatrix} + Z\{b'd\}\begin{pmatrix}N\\ M\end{pmatrix}\right]\\ &+ \left[\{ab\}\begin{pmatrix}NT\\ MT\end{pmatrix} + Z\{b'd\}\begin{pmatrix}NT\\ MT\end{pmatrix}\right] - \begin{pmatrix}\alpha_x\\ \alpha_y\\ \alpha_{xy}\end{pmatrix}\Delta T\Bigg\}\end{aligned}\right\} \tag{40}$$

Since all the mechanical loads are based on N_x and follows the ratio k1, the Eqn 40 can be rewritten as,

$$\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\end{pmatrix} = \overline{Q_{\iota J}}\left\{\left[\{ab\}\begin{pmatrix}1\\ k1\\ 0\\ 0\\ 0\\ 0\end{pmatrix} + Z\{b'd\}\begin{pmatrix}1\\ k1\\ 0\\ 0\\ 0\\ 0\end{pmatrix}\right]N_x + \left[\{ab\}\begin{pmatrix}NT\\ MT\end{pmatrix} + Z\{b'd\}\begin{pmatrix}NT\\ MT\end{pmatrix}\right] - \begin{pmatrix}\alpha_x\\ \alpha_y\\ \alpha_{xy}\end{pmatrix}\Delta T\right\}$$

$$\left.\begin{aligned}\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\end{pmatrix} = \overline{Q_{\iota J}}&\left\{\left[\{ab\}\begin{pmatrix}1\\ k1\\ 0\\ 0\\ 0\\ 0\end{pmatrix} + Z\{b'd\}\begin{pmatrix}1\\ k1\\ 0\\ 0\\ 0\\ 0\end{pmatrix}\right]N_x\right\}\\ &+ \overline{Q_{\iota J}}\left[\{ab\}\begin{pmatrix}NT\\ MT\end{pmatrix} + Z\{b'd\}\begin{pmatrix}NT\\ MT\end{pmatrix}\right] - \overline{Q_{\iota J}}\begin{pmatrix}\alpha_x\\ \alpha_y\\ \alpha_{xy}\end{pmatrix}\Delta T\end{aligned}\right\} \tag{41}$$

From the Eqn 41, the stresses in the outer (0° Ply) and inner (60° Ply) layers can be shown to be

$$\left.\begin{aligned} \sigma_1^{(1)} &= 1.8357\left(\frac{N_x}{t}\right) + 48634 * \Delta T \\ \sigma_2^{(1)} &= 0.0890\left(\frac{N_x}{t}\right) - 75099 * \Delta T \\ \sigma_{12}^{(1)} &= 0 \end{aligned}\right\} \qquad (42)$$

$$\left.\begin{aligned} \sigma_1^{(2)} &= 0.8544\left(\frac{N_x}{t}\right) - 98127 * \Delta T \\ \sigma_2^{(2)} &= 0.1521\left(\frac{N_x}{t}\right) + 78279 * \Delta T \\ \sigma_{12}^{(2)} &= -0.0526\left(\frac{N_x}{t}\right) + 2654 * \Delta T \end{aligned}\right\} \qquad (43)$$

The stresses have now been determined as a linear function of the applied loads, Nx and ΔT. Note that the laminate stresses have been expressed as a function of the average laminate stress, Nx/t. The stresses in the laminae are different from one another because of different fiber orientation of each lamina.

Application of the Laminate Failure Criterion

A failure criterion must be applied to determine the maximum value of Nx/t that can be sustained without failure of any layer. Actually, the failure criterion is applied to each layer separately. The Yeh-Stratton failure criterion from the Eqn 29 for each layer can be expressed as

$$\frac{\sigma_1}{F_1} + \frac{\sigma_2}{F_2} + B_{12}\sigma_1\sigma_2 + \frac{\tau_{12}^2}{F_6^2} = 1$$

If ΔT is zero, which means curing temperature and operational temperature are same.

Upon substitution of the stresses from Eqns 42 and 43 in to the Y-S Criterion, results a quadratic equation with solution.

For the outer (0^o ply) layer

$$-2.003 * 10^{-18} * \left(\frac{N_x}{t}\right)^2 + 2.934 * 10^{-9} * \frac{N_x}{t} - 1 = 0 \quad (44)$$

By solving the quadratic equation, we get two solutions for Nx/t

$$\frac{N_x}{t} = 9.2608 * 10^8 \; or \; 5.392 * 10^8 \quad (45)$$

To be conservative, the lesser value is chosen as the failure load value.

For the inner (60^o ply) layer

$$-1.292 * 10^{-18} * \left(\frac{N_x}{t}\right)^2 + 3.671 * 10^{-9} * \frac{N_x}{t} - 1 = 0 \quad (46)$$

By solving the quadratic equation, we get two solutions for Nx/t

$$\frac{N_x}{t} = 2.5364 * 10^9 \; or \; 3.052 * 10^8 \quad (47)$$

Again the lesser value is chosen as the failure load value.

As 3.052e2 MPa is the lowest of all the values, we know that the inner (60^o ply) layer is failing first.

In order to find whether it was a fibre direction failure or transverse (perpendicular to the directions of fibres) direction failure, we have to apply the load in the failure criterion.

By choosing $\frac{N_x}{t} = 3.052 * 10^8$, in the inner ($60^o$ ply) layer, we get $\frac{\sigma_1}{F_1} = 0.1534$ $\frac{\sigma_2}{F_2} = 0.9669$. This ratio between applied stress and allowable stress makes it clear that failure is in transverse direction.

Hence we can conclude that the inner (60^o ply) layer has failed in the transverse direction but, still can take load on the longitudinal/fibre direction.

Now the reduced stiffness matrices for the lamina will change to

$$Q_{ij}^{(1)} = \begin{pmatrix} 1.2795 & 0.0327 & 0 \\ 0.0327 & 0.1128 & 0 \\ 0 & 0 & 0.0579 \end{pmatrix} * 10^{11} \qquad (48)$$

$$Q_{ij}^{(2)} = \begin{pmatrix} 1.2795 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} * 10^{11} \qquad (49)$$

$$\overline{Q_{ij}}^{(1)} = \begin{pmatrix} 1.2795 & 0.0327 & 0 \\ 0.0327 & 0.1128 & 0 \\ 0 & 0 & 0.0579 \end{pmatrix} * 10^{11} \qquad (50)$$

$$\overline{Q_{ij}}^{(2)} = \begin{pmatrix} 0.7997 & 2.3990 & 1.3851 \\ 2.3990 & 7.1971 & 4.1553 \\ 1.3851 & 4.1553 & 2.3990 \end{pmatrix} * 10^{10} \qquad (51)$$

The laminate extensional stiffness matrix is

$$ABD = \begin{pmatrix} 1.1103 & 0.2824 & 0 & 0 & 0 & 0 \\ 0.2805 & 0.8590 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.3026 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4602 & 0.0586 & 0.0158 \\ 0 & 0 & 0 & 0.0586 & 0.1931 & 0.0475 \\ 0 & 0 & 0 & 0.0158 & 0.0475 & 0.0673 \end{pmatrix} * 10^{11} \qquad (52)$$

The inverse extensional stiffness matrix is

$$abd = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0227 & -0.0073 & -0.0002 \\ 0 & 0 & 0 & -0.0073 & 0.0702 & -0.0478 \\ 0 & 0 & 0 & -0.0002 & -0.0478 & 0.1824 \end{pmatrix} \qquad (53)$$

Now, to double check whether the changed ABD matrix causes any more consequent and instantaneous failures, present stress values will be calculated using the calculated Nx/t and it should be applied in the criterion again. Because the change in the ABD matrix will surely cause sudden change in the stress values in the individual lamina, which might cause some of the laminae to fail instantly after the first ply failure.

The stress values are,

$$\left.\begin{aligned} \sigma_1^{(1)} &= 6.2645 * 10^8 \\ \sigma_2^{(1)} &= 2.784 * 10^7 \\ \sigma_{12}^{(1)} &= 0 \end{aligned}\right\} \tag{54}$$

$$\left.\begin{aligned} \sigma_1^{(2)} &= 2.57 * 10^8 \\ \sigma_2^{(2)} &= 0 \\ \sigma_{12}^{(2)} &= 0 \end{aligned}\right\} \tag{55}$$

Since the above values of stresses do not exceed the material allowable stress limits, no consecutive failures will occur.

Again by using the Equation 41 and new ABD matrices from Equations 52 and 53, the stresses in the outer (0^o ply) and inner (60^o ply) layers can be shown in a linear equation as,

$$\left.\begin{aligned} \sigma_1^{(1)} &= 2.0526\left(\frac{N_x}{t}\right) - 26583 * \Delta T \\ \sigma_2^{(1)} &= 0.0912\left(\frac{N_x}{t}\right) - 79750 * \Delta T \\ \sigma_{12}^{(1)} &= 0 \end{aligned}\right\} \tag{56}$$

$$\left.\begin{aligned}\sigma_1^{(2)} &= 0.8421\left(\frac{N_x}{t}\right) + 79750 * \Delta T \\ \sigma_2^{(2)} &= 0 \\ \sigma_{12}^{(2)} &= 0\end{aligned}\right\} \tag{57}$$

From which, upon substitution of the stresses in the Y-S Criterion and subtraction of the stresses from the already applied and existing load, results a quadratic equation with solution.

For the outer (0° ply) layer,

$$-2.295 * 10^{-18} * \left(\frac{N_x}{t}\right)^2 + 1.707 * 10^{-9} * \frac{N_x}{t} - 0.2652 = 0 \tag{58}$$

By solving the quadratic equation, we get two solutions for Nx/t

$$\frac{N_x}{t} = 2.21 * 10^8 \ or \ 5.23 * 10^8 \tag{59}$$

The lesser value is chosen as the failure load value.

For the inner (60° ply) layer,

$$-5.7288 * 10^{-34} * \left(\frac{N_x}{t}\right)^2 + 4.95 * 10^{-10} * \frac{N_x}{t} - 0.8488 = 0 \tag{60}$$

By solving the quadratic equation, we get two solutions for Nx/t

$$\frac{N_x}{t} = 8.6469 * 10^{23} \ or \ 1.7146 * 10^9 \tag{61}$$

The lesser value is chosen as the failure load value. Note that the Present calculated stress value is an additional stress value to the stress value calculated in the previous run.

As 2.21e2 Mpa is the lowest of all the values, we know that the outer (0° ply) layer is failing second.

Now the total present stress is 2.21e2 + 3.052e2=5.262e2.

After applying the stress values in to Y-S criterion for the outer (0^o ply) layer, it is found that $\frac{\sigma_1}{F_1} = 0.6354\ and\ \frac{\sigma_2}{F_2} = 1.0$. It means transverse direction failure has happened in the outer (0^o ply) layer.

Now, the reduced stiffness matrix shall be changed to

$$Q_{ij}^{(1)} = \begin{pmatrix} 1.2795 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} * 10^{11} \tag{62}$$

$$Q_{ij}^{(2)} = \begin{pmatrix} 1.2795 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} * 10^{11} \tag{63}$$

$$\overline{Q_{ij}}^{(1)} = \begin{pmatrix} 1.2795 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} * 10^{11} \tag{64}$$

$$\overline{Q_{ij}}^{(2)} = \begin{pmatrix} 0.7997 & 2.3990 & 1.3851 \\ 2.3990 & 7.1971 & 4.1553 \\ 1.3851 & 4.1553 & 2.3990 \end{pmatrix} * 10^{10} \tag{65}$$

The laminate extensional stiffness matrix is

$$ABD = \begin{pmatrix} 1.1103 & 0.2562 & 0 & 0 & 0 & 0 \\ 0.2562 & 0.7687 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.2562 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} * 10^{8} \tag{66}$$

The inverse extensional stiffness matrix is

$$abd = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0225 & -0.0075 & 0 \\ 0 & 0 & 0 & -0.0075 & 0.1091 & -0.1072 \\ 0 & 0 & 0 & 0 & -0.1072 & 0.3197 \end{pmatrix} \tag{67}$$

Again no consecutive failures have been found.

Again by using the Equation 41 and new ABD matrices from Equations 66 and 67, the stresses in the outer (0^o ply) and inner (60^o ply) layers can be shown in a linear equation as,

$$\left.\begin{aligned}\sigma_1^{(1)} &= 2.0222\left(\frac{N_x}{t}\right) - 0.5239 * 10^{-9} * \Delta T \\ \sigma_2^{(1)} &= 0 \\ \sigma_{12}^{(1)} &= 0\end{aligned}\right\} \tag{68}$$

$$\left.\begin{aligned}\sigma_1^{(2)} &= 0.9333\left(\frac{N_x}{t}\right) - 0.2107 * 10^{-9} * \Delta T \\ \sigma_2^{(2)} &= 0 \\ \sigma_{12}^{(2)} &= 0\end{aligned}\right\} \tag{69}$$

From which, upon substitution of the stresses in the Y-S Criterion and subtraction of the stresses from the already applied and existing load, results a quadratic equation with solution.

For the outer (0^o ply) layer

$$1.19 * 10^{-9} * \frac{N_x}{t} - 0.3741 = 0 \tag{70}$$

By solving the quadratic equation, we get one solution for Nx/t

$$\frac{N_x}{t} = 3.145 * 10^8 \tag{71}$$

For the outer (0^o ply) layer

$$5.49 * 10^{-10} * \frac{N_x}{t} - 0.7111 = 0 \tag{72}$$

By solving the quadratic equation, we get one solution for Nx/t

$$\frac{N_x}{t} = 1.3028 * 10^9 \tag{73}$$

The lesser value is chosen as the failure load value.

As 3.145e2 Mpa is the lowest of all the values, we know that the outer (0^o ply) layer is failing again. It is found that $\frac{\sigma_1}{F_1} = 1.0\ and\ \frac{\sigma_2}{F_2} = 0$. It means fiber failure has happened in the outer (0^o ply) layer. So the ultimate failure load per unit thickness of the laminate according to Y-S criterion is at Nx/t= 2.21e2 + 3.052e2 + 3.145e2 = 841 Mpa for the ration of $\frac{N_y}{N_x} = 0.4$

CHAPTER 7

RESULTS

The Yeh-Stratton criterion was utilized in predicting composite failures of experimental tests that were carried out on a $[0/\pm 60]_s$ laminate. Bi-axial stress conditions were imposed in tubular form by combinations of internal pressure and axial load [15]. The laminate is assumed to have reached ultimate failure once the fiber in the plies have failed according to the criteria. As it can be seen in Figure 5 in Appendix C, the failure predicted by the Y-S criterion fits the experimental results very well.

Other failure criteria are also used for predicting the composite failure and compared in Figure 6 in Appendix C. The Hashin polynomial predicted failure at a bit higher load comparing to the experimental results. The Tsai-Wu quadratic stress polynomial being conservative by large factors did not agree well with the experimental results. As it can be seen in Figure 6 and Figure 7 in Appendix C, only the Yeh-Stratton criterion and the Maximum Stress criterion gave a good correlation with the experimental data. This may be because three terms in the Y-S criterion are representative of the Maximum Stress criterion.

Unfortunately, the failure data for axial-to-hoop stress ratio (k1) greater than 1.0 were not available. For a future study, it is recommended that testing be done in the region where k1>1.

APPENDICES

APPENDIX A

EXPERIMENTAL SETUP

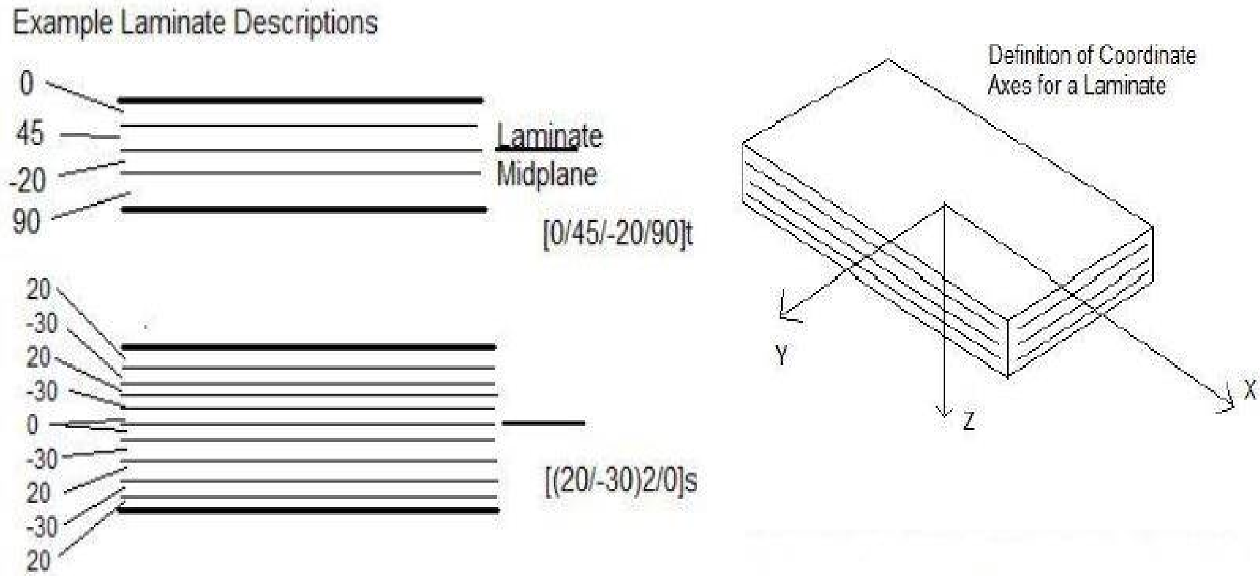

FIGURE 1. Composite laminates, lay-up nomenclature [24].

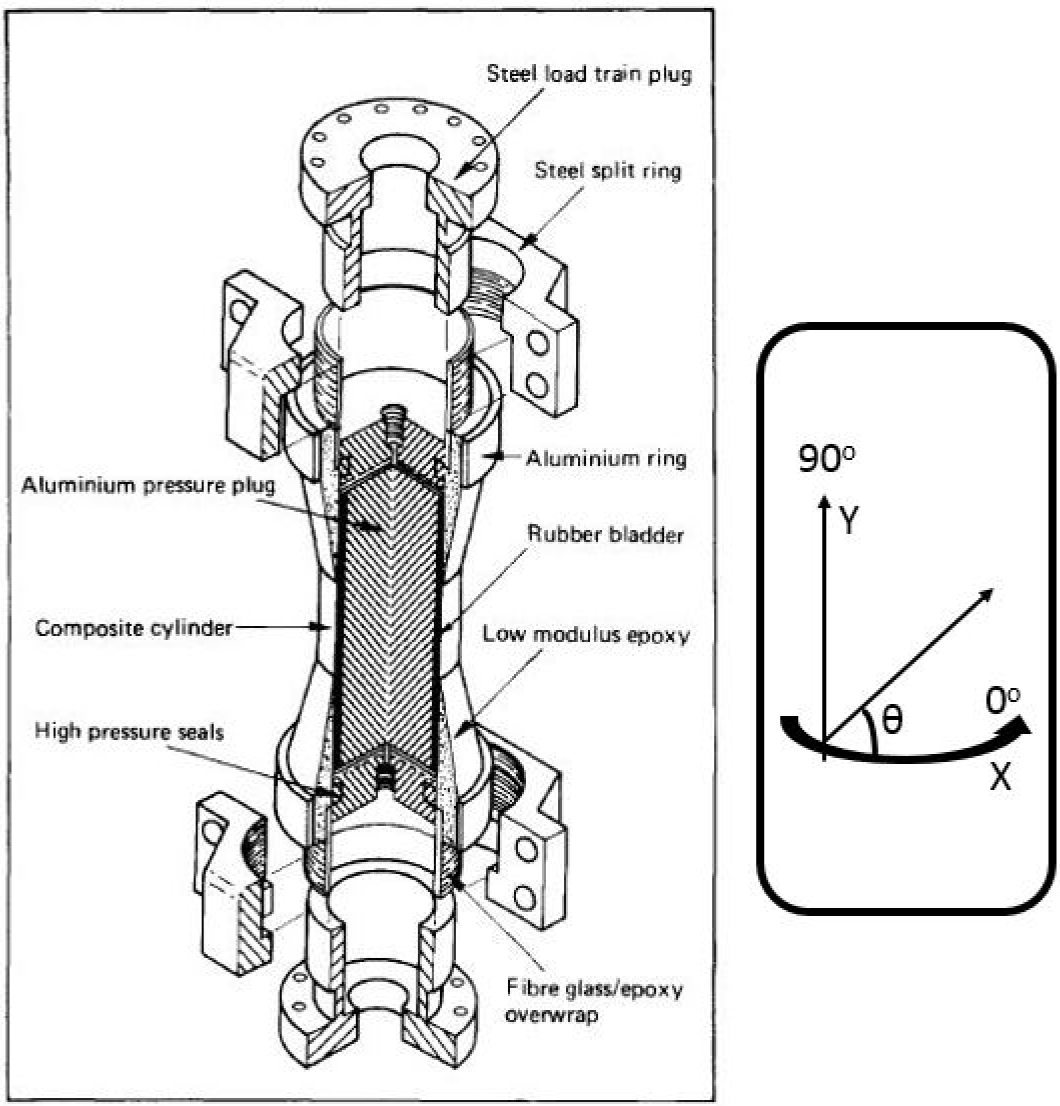

FIGURE 2. Schematic of the four inch tubular specimen with end grips and internal pressure plug [15].

TABLE 1. Fiber Lot Acceptance Data for AS4/3501-6 Carbon/Epoxy [15].

Property	Mean Value	Coefficient of Variation
Tensile Strength, X_T	1990 Mpa (289 ksi)	6.1%
Fiber Strain	1.384%	4.6%

TABLE 2. Measured Strength Properties of Tubular [0/60/-60/0/-60/60/0] Carbon/Epoxy Specimens [15].

Specimen number	Stresses at failure (Mpa)	
	Axial Stress, σ_z	Hoop Stress, σ_θ
LTCU-86-9-#2	17.2	749.3
LTCU-86-9-#3	22.4	769.5
LTCU-86-10-#2	533.5	1066.9
LTCU-86-10-#3	425.6	851.1
LTCU-86-11-#1	229.6	839.8
LTCU-86-11-#2	677.8	1002.3
LTCU-86-11-#3	462.7	925.3
LTCU-86-12-#2	629.7	921.9
LTCU-86-12-#3	238.6	969.4

TABLE 3. Material Properties Used in Failure Analysis of AS4/3501-6 [15].

Property	Value	Source
Fiber direction ply strength in tension(F_{1t})	1990 MPa	Ref. 15
F_{1t} modified for use in linear analysis(F_{1t})	1758 MPa	Ref. 15
Fiber direction ply strength in compression(F_{1c})	-1193 MPa	Ref. 17
Transverse normal strength in tension(F_{2t})	48 MPa	Ref. 19
Transverse normal strength in compression(F_{2c})	168 MPa	Ref. 19
In-plane shear strength (F_6)	96 MPa	Ref. 18

TABLE 4. Geometrical Properties of the [0/60/-60/0/-60/60/0] Laminate [15].

Ply No	Ply Orientation(deg)	Thickness of the Ply(mm)
1	0	.267
2	60	.267
3	-60	.267
4	0	.267
5	-60	.267
6	60	.267
7	0	.267

TABLE 5. Material Properties of the [0/60/-60/0/-60/60/0] Laminate [15].

Property	Value
E1	1.27E+11 Pa
E2	1.12E+10 Pa
G12	5.79E+9 Pa
ν_{12}	0.29
ν_{21}	2.56E-2
α_1	3.50E-6 / ^{o}F
α_2	1.14E-5 / ^{o}F
β_1	0 in/in/g/g
β_2	0 in/in/g/g
F1t	1.70E+9 Pa
F2t	4.80E+7 Pa
F1c	1.19E+9 Pa
F2c	1.68E+8 Pa
F6	9.60E+7 Pa

APPENDIX B

FLOWCHART

Flowchart of the MATLAB Algorithm:

Input the Properties of the Laminate

Calculate the ABD Matrix of the Laminate

$$(\sigma_{1,2,12} = Const\_of\_laminate * \left(\frac{N_x}{t}\right)) + stress\ due\ to\ previous\ load$$

Find the Const_of_laminate, using ABD matrix and load ratio which is (N_y/N_x) *, stress due to previous load will initially be zero, and for the next run it will be taken from the previous run of the program.*

Use the respective failure criterion to find (N_x/t) *,Which is first ply failure load per unit thickness*

Change Q_{12} *by applying the criterion and stress values, and get* $(Q_{12})_2$

Find $(ABD)_2$ *, which is ABD after first ply failure*

Find $(\sigma_{1,2,12})_2$, which is stress on the layups after first ply failure.

Apply the stress values in to the failure criterion.

Check for more layer failures at same load.

Find $(Q_{12})_3$, which is Q matrix after more failure at same load instantly after first ply failure, mostly $(Q_{12})_3=(Q_{12})_2$ because, generally no istant failures follow the first failure at the same load as the first ply failure.

Find $(ABD)_3$,

similar to $(Q_{12})_3$, Mostly $(ABD)_3 = (ABD)_2$

Find $(\sigma_{1,2,12})_3$,

similar to $(Q_{12})_3$, Mostly $(\sigma_{1,2,12})_3 = (\sigma_{1,2,12})_2$

Stress due to previous load (Input back to

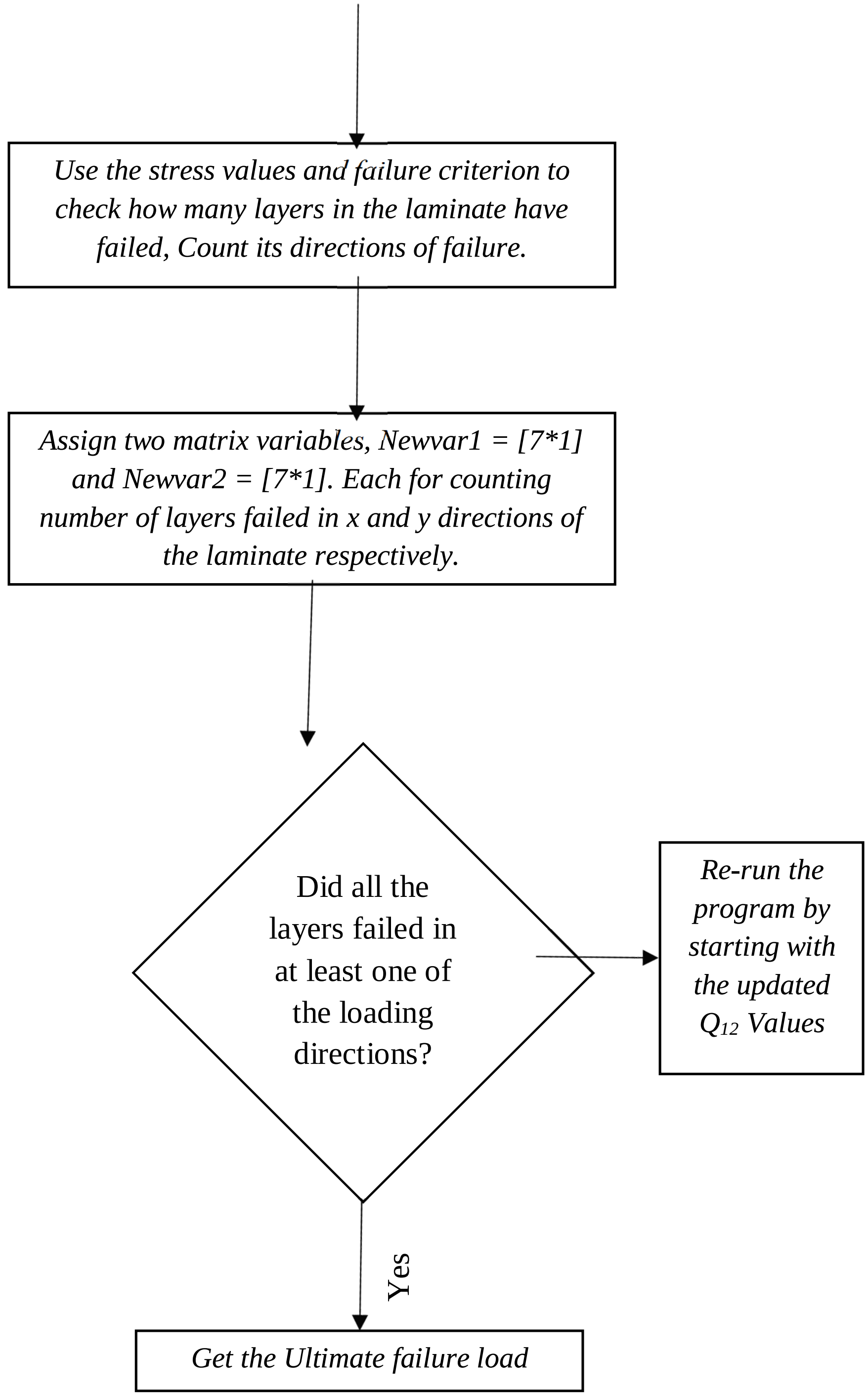

FIGURE 3. Flowchart of MATLAB Algorithm.

APPENDIX C

PLOT RESULTS

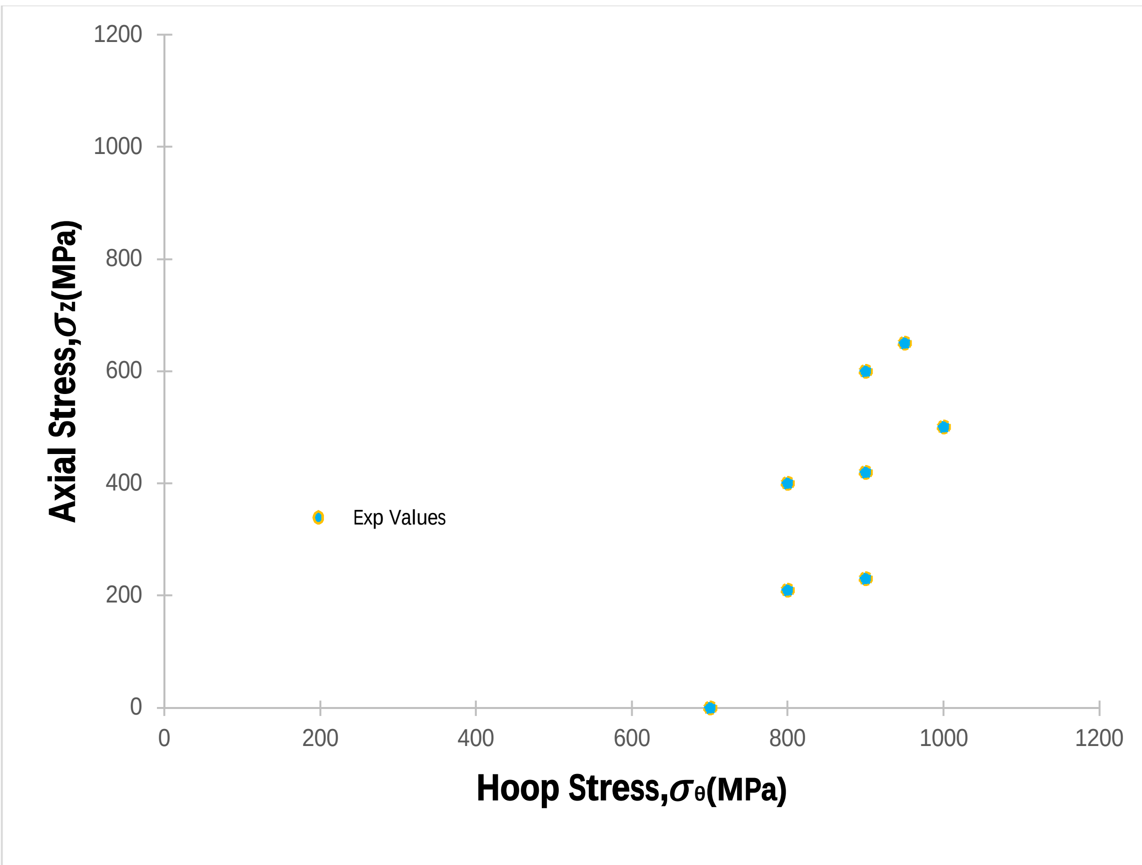

FIGURE 4. Comparison of hoop and axial stresses at failure for a carbon/epoxy laminate with a stacking sequence of [0/60/-60/0/-60/60/0] [15].

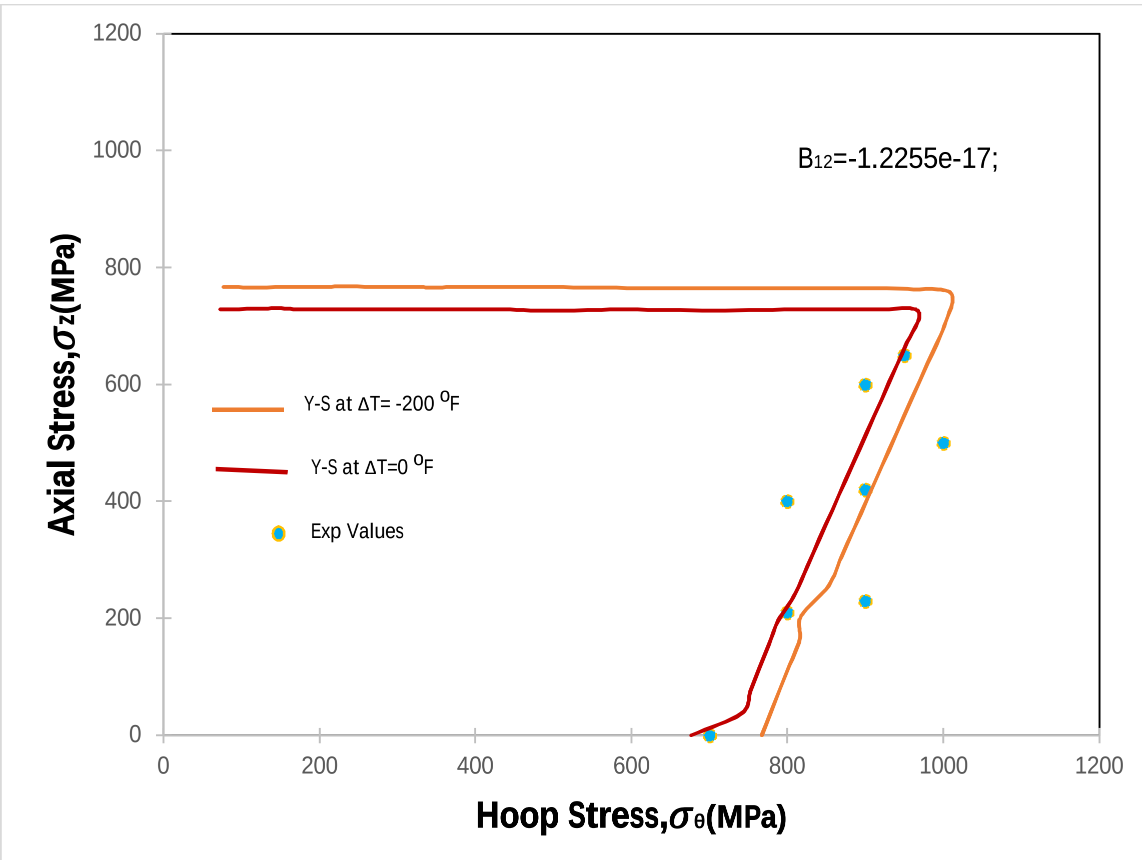

FIGURE 5. Comparison of stresses at failure for a carbon/epoxy laminate with a stacking sequence of [0/60/-60/0/-60/60/0] by y-s criterion at δt=-200ºf and at δt=0ºf.

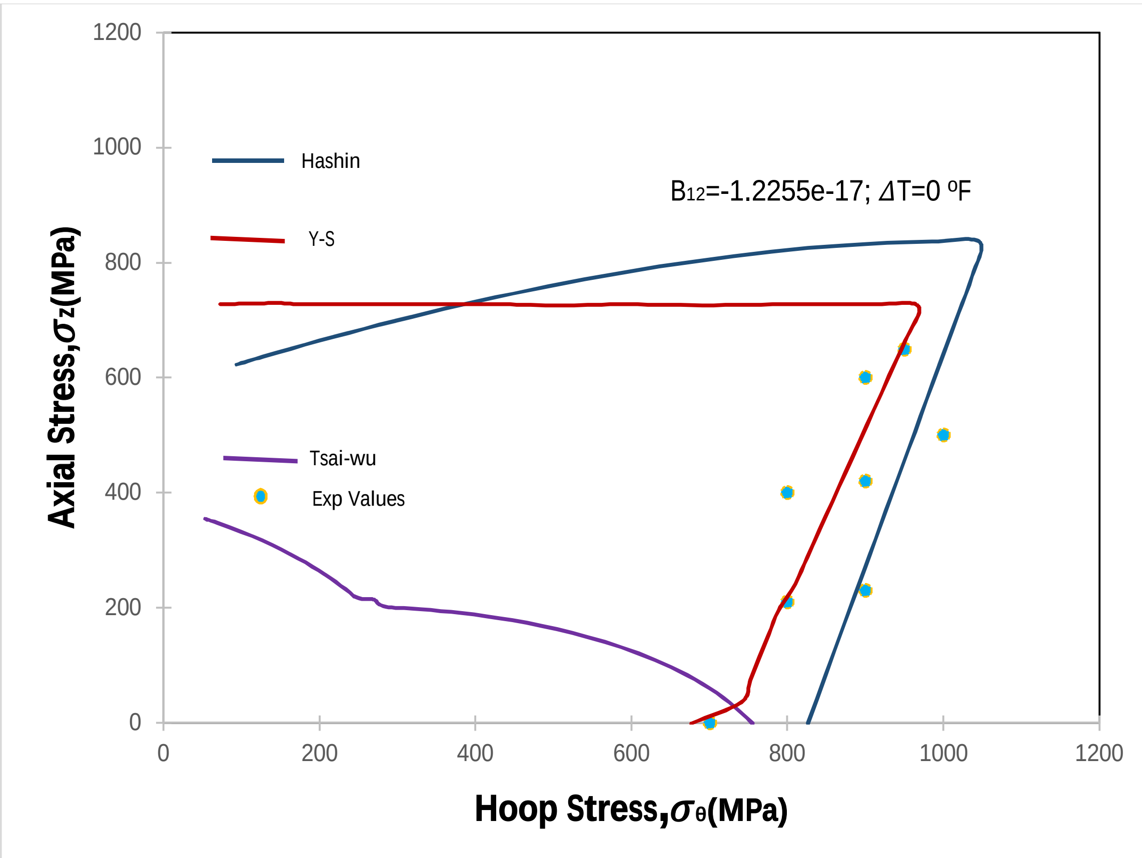

FIGURE 6. Comparison of stresses at failure for a carbon/epoxy laminate with a stacking sequence of [0/60/-60/0/-60/60/0] by tsai-wu stress polynomial, hashin failure criterion and yeh-stratton criterion.

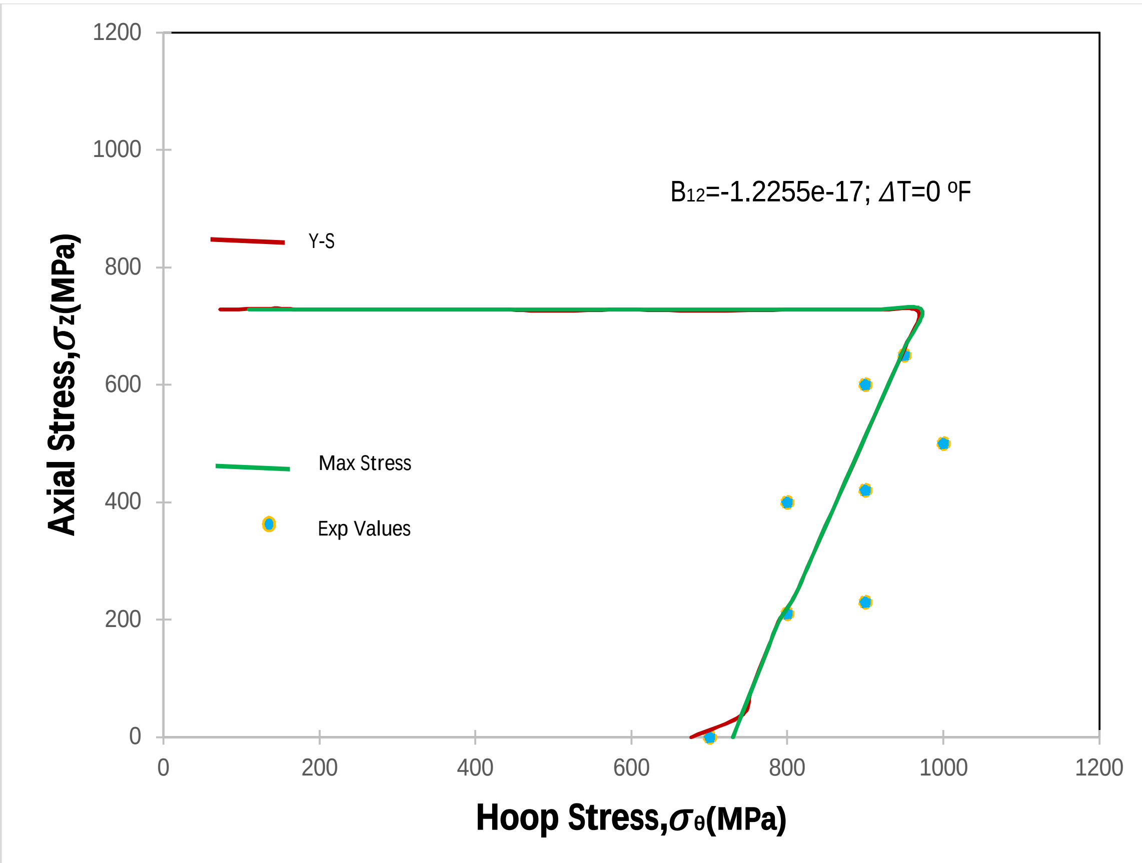

FIGURE 7. Comparison of stresses at failure for a carbon/epoxy laminate with a stacking sequence of [0/60/-60/0/-60/60/0] by maximum stress and yeh-stratton criterion.

APPENDIX D

MATLAB CODE

Yeh-Stratton Criterion

```
colorstring = 'kbgry';
figure(1); cla;
hold on
B12_R=1;

B12=-1.22549019607843e-17;

Y_S=repmat(0.5,(size(t,1)),2);


RR=1;

k1=0;
k1_x=1;
k1_intervals=0.05;
while k1<20

   if k1<=1
       Nx=8e8;
   else
       Nx=4e8;
   end


Nxtn=zeros;
s=2;
R=2;
    for i=1:size(t,1)

        Q12{i} = [(E1(i,1)/(1-v12(i,1)*v21(i,1)))...
            (v21(i,1)*E1(i,1)/(1-v12(i,1)*v21(i,1)))  0; ...
             (v21(i,1)*E1(i,1)/(1-v12(i,1)*v21(i,1))) ...
             (E2(i,1)/(1-v12(i,1)*v21(i,1)))  0; ...
               0   0   G12(i,1)];
    end

    newvar1 = ones(size(t,1),1);
    newvar2 = ones(size(t,1),1);

    while s>1
            NT=zeros;
            NH=zeros;
            Ny=k1*Nx;
            Nxy=k2*Nx;
            Mx=k3*Nx;
```

```
            My=k4*Nx;
            Mxy=k5*Nx;
            N = [Nx ; Ny ; Nxy];
            M = [Mx ; My ; Mxy];

            A=0;B=0;D=0;NT=0;NH=0;MT=0;MH=0;
            ABD=0;
            NT_DelT=0;NH_DelT=0;MT_DelT=0;MH_DelT=0;

            %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
            %%% Setting the (h) Matrix  %%%
            %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

            x=zeros(size(t,1)+1,1);
            x(1,1)=sum(t)/2;
            for i=2:size(t,1)+1
                x(i)=x(i-1)-t(i-1);
            end
            h=flipud(x);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%%% Calculating (Qbar)'s, (A,B,& D) Matrices,NT, NH, MT, & MH
%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%

            k=1;
            for i=1:size(t,1)
           Alpha = [Alpha1(i);Alpha2(i);0];
           Beta =  [Beta1(i);Beta2(i);0];
           S12{i} =[1/E1(i,1)             -v12(i,1)/E1(i,1)   0;
...
                  -v12(i,1)/E1(i,1)      1/E2(i,1)          0;
...
              0                        0
1/G12(i,1)];

      T_sig{i} =[(cos(th(i,1)))^2  (sin(th(i,1)))^2 ...
      (2*cos(th(i,1))*sin(th(i,1))); ...
   (sin(th(i,1)))^2  (cos(th(i,1)))^2 -
2*cos(th(i,1))*sin(th(i,1)); ...
      -1*cos(th(i,1))*sin(th(i,1))  1*cos(th(i,1))*sin(th(i,1))
...
      ((cos(th(i,1)))^2)-((sin(th(i,1)))^2)];
                T_sig_in{i} = inv(T_sig{i});
      T_eps{i} = [(cos(th(i,1)))^2  (sin(th(i,1)))^2 ...
```

```
      (1*cos(th(i,1))*sin(th(i,1))); ...
   (sin(th(i,1)))^2  (cos(th(i,1)))^2 -
1*cos(th(i,1))*sin(th(i,1)); ...
     -2*cos(th(i,1))*sin(th(i,1))  2*cos(th(i,1))*sin(th(i,1))
...
     ((cos(th(i,1)))^2)-((sin(th(i,1)))^2)];
                T_eps_in{i} = inv(T_eps{i});


                Qbar{i} =T_sig_in{i} * Q12{i} * T_eps{i};
                Sbar{i} = inv(Qbar{i});

                A = (A + Qbar{i} * (h(i+1,1) - h(i,1)));
                B = (B + 1/2 * Qbar{i} * (h(i+1,1)^2 -
h(i,1)^2));
                D = (D + 1/3 * Qbar{i} * (h(i+1,1)^3 -
h(i,1)^3));

                Alpha_xy{i} = T_sig_in{i}*Alpha;
                Beta_xy{i} = T_sig_in{i}*Beta;


%To change eps shear strain to gamma shear strain

            Alpha_xy_2eps{i}=Alpha_xy{i};
            Beta_xy_2eps{i}=Beta_xy{i};
            Alpha_xy_2eps{i}(3,1)=2*Alpha_xy_2eps{i}(3,1);
            Beta_xy_2eps{i}(3,1)=2*Beta_xy_2eps{i}(3,1);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%



                NTp(:,k) = (DelT * Qbar{i} * T_sig_in{i} * Alpha
*...
                    (h(i+1,1) - h(i,1)));
                NHp(:,k) = (DelC * Qbar{i} * T_sig_in{i} * Beta
*...
                    (h(i,1) - h(i+1,1)));
            MTp(:,k) = (1/2 * DelT * Qbar{i} * T_sig_in{i} *
Alpha...
                    * (h(i,1)^2 - h(i+1,1)^2));
            MHp(:,k) = (1/2 * DelC * Qbar{i} * T_sig_in{i} *
Beta...
                    * (h(i,1)^2 - h(i+1,1)^2));
```

```
                NT = (NT + DelT * Qbar{i} *
(Alpha_xy_2eps{i})*...
                    (h(i+1,1) - h(i,1)));
                NH = (NH + DelC * Qbar{i} * (Beta_xy_2eps{i})
*...
                    (h(i+1,1) - h(i,1)));
             MT = (MT + 1/2 * DelT * Qbar{i} *
(Alpha_xy_2eps{i}) *...
                    (h(i+1,1)^2 - h(i,1)^2));
              MH = (MH + 1/2 * DelC * Qbar{i} *
(Beta_xy_2eps{i}) *...
                    (h(i+1,1)^2 - h(i,1)^2));

                NT_DelT = (NT_DelT + Qbar{i} *
(Alpha_xy_2eps{i})*...
                    (h(i+1,1) - h(i,1)));
                NH_DelT = (NH_DelT + Qbar{i} * (Beta_xy_2eps{i})
*...
                    (h(i+1,1) - h(i,1)));
           MT_DelT = (MT_DelT + 1/2 * Qbar{i} *
(Alpha_xy_2eps{i}) *...
                  (h(i+1,1)^2 - h(i,1)^2));
           MH_DelT = (MH_DelT + 1/2 * Qbar{i} *
(Beta_xy_2eps{i}) *...
                  (h(i+1,1)^2 - h(i,1)^2));

                k=k+1;
            end

            NT_MT_by_th =  ([NT;MT]/sum(t));
            NT_MT_by_th_DelT = ([NT_DelT;MT_DelT]/(sum(t)));


            Nbar = (N + NT + NH);
            Mbar = (M + MT + MH);
            ABD = ([A B;B D]);
            N_M=([Nbar;Mbar]);
            Load_Coeff = ([1;k1;k2;k3;k4;k5]);
            abd = pinv(ABD);
            a = (abd(1:3,1:3));
            b = (abd(1:3,4:6));
            bT= (abd(4:6,1:3));
            d = (abd(4:6,4:6));
            ab = ([a b]);
            record_of_Qbar11(R,:)=Qbar{1}(1,:);
            record_of_Qbar21(R,:)=Qbar{2}(1,:);
            record_of_Qbar22(R,:)=Qbar{2}(2,:);
```

```
            record_of_Qbar23(R,:)=Qbar{2}(3,:);
            record_of_Qbar31(R,:)=Qbar{3}(1,:);
            record_of_Q12(R,:)=Q12{1}(1,:);

        curv_mat=zeros(size(t,1),1);
        curv_mat(1,1)=0.5*sum(t,1)-0.5*t(1,1);
        for i=2:size(t,1)
        curv_mat(i,1) = curv_mat(i-1,1)-0.5*(t(i-1,1)+t(i,1));
        end


        for i=1:size(t,1)
            sig_a_f{1,i}=zeros(3,1);
            sig_a_f{2,i}=zeros(3,1);

        end

    if DelT==0
    for i = 1:size(t,1)
  sigxy_FT{i} =
((Qbar{i}*(ab*Load_Coeff))+(curv_mat(i,1)*Qbar{i}...
       *([bT d]*Load_Coeff)))*sum(t);
  sigxy_ST{i} =
((Qbar{i}*(((ab*[NT_DelT;MT_DelT]))+(curv_mat(i,1)*...
  ([bT d]*[NT_DelT;MT_DelT])))))...
  -(Qbar{i}*(Alpha_xy_2eps{i})*DelTzero))/DelTzero;
    end
    for i = 1:size(t,1)
    sig12_FT{i} = (T_sig{i}*sigxy_FT{i});
    sig12_ST{i} = (T_sig{i}*sigxy_ST{i});
    end
    else
            for i = 1:size(t,1)
      sigxy_FT{i} =
((Qbar{i}*(ab*Load_Coeff))+(curv_mat(i,1)*Qbar{i}...
                *([bT d]*Load_Coeff)))*sum(t);
           sigxy_ST{i} =
((Qbar{i}*(((ab*[NT;MT]))+(curv_mat(i,1)*...
      ([bT d]*[NT;MT]))))-
(Qbar{i}*(Alpha_xy_2eps{i})*DelT))/DelT;
            end
            for i = 1:size(t,1)
            sig12_FT{i} = (T_sig{i}*sigxy_FT{i});
            sig12_ST{i} = (T_sig{i}*sigxy_ST{i});
            end
    end
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% Setting the (z) Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

z(1,1)=sum(t)/2;
for i=2:size(t,1)
    z(i-1,1)=z(i-1);
    z(i-1,2)=z(i-1)-t(i-1);
    z(i,1) = z(i-1,2);
end
z(size(t,1),2) = -sum(t)/2;
x=flipud(z);
x(:,[1,2])=x(:,[2,1]);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Calculating [sig]k & [eps]k for each ply %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

eps0_k = (abd * [Nbar ; Mbar]);
eps0 = (eps0_k(1:3));
k    = (eps0_k(4:6));

for i = 1:size(t,1)
eps{i,1} = (eps0 + curv_mat(i)*k);
eps{i,2} = (eps0 + x(i,2)*k);
end
for i = 1:size(t,1)
sigxy{i,1} = (Qbar{i}*(eps{i,1}-(DelT*
Alpha_xy_2eps{i})));
sigxy{i,2} = (Qbar{i}*(eps{i,2}-(DelT*
Alpha_xy_2eps{i})));
end

for i = 1:size(t,1)
sig{i,1} = (T_sig{i}*sigxy{i,1});
sig{i,2} = (T_sig{i}*sigxy{i,2});
end



%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%             calculating Nxt              %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


for i=1:size(t,1)
```

```
            if (sig{i,1}(1,1)>=0)
            F1(i)=F1t;
            else
            F1(i)=F1c;
            end
            if (sig{i,1}(2,1)>=0)
            F2(i)=F2t;
            else
            F2(i)=F2c;
            end
          end

              for i=1:size(t,1)

    Quadratic_a(i)=(sig12_FT{i}(1,1)*sig12_FT{i}(2,1)*B12)+...
        (sig12_FT{i}(3,1)^2/F6^2);

Quadratic_b(i)=(sig12_FT{i}(1,1)/F1(i))+(sig12_FT{i}(2,1)/F2(i))
+...
        (sig12_FT{i}(1,1)*sig12_ST{i}(2,1)*DelT*B12)+...
        (sig12_FT{i}(2,1)*sig12_ST{i}(1,1)*DelT*B12)+...
        (2*sig12_FT{i}(3,1)*sig12_ST{i}(3,1)*DelT)/(F6^2)+...
        (sig12_FT{i}(1,1)*sig_a_f{R,i}(2,1)*B12)+...
        (sig12_FT{i}(2,1)*sig_a_f{R,i}(1,1)*B12)+...
        ((2*sig12_FT{i}(3,1)*sig_a_f{R,i}(3,1))/(F6^2));
    Quadratic_c(i)=(((sig12_ST{i}(1,1)*DelT)/F1(i))+...
        ((sig12_ST{i}(2,1)*DelT)/F2(i))+...
        (sig12_ST{i}(1,1)*sig12_ST{i}(2,1)*DelT*DelT*B12)+...

(((sig12_ST{i}(3,1)*DelT)^2)/(F6^2))+(sig_a_f{R,i}(1,1)/F1(i))+.
..
        (sig_a_f{R,i}(2,1)/F2(i))+...
        (sig_a_f{R,i}(1,1)*sig_a_f{R,i}(2,1)*B12)+...
        ((sig_a_f{R,i}(3,1)^2)/(F6^2))+...
        (sig_a_f{R,i}(1,1)*sig12_ST{i}(2,1)*DelT*B12)+...
        (sig_a_f{R,i}(2,1)*sig12_ST{i}(1,1)*DelT*B12)+...
        ((2*sig_a_f{R,i}(3,1)*sig12_ST{i}(3,1))*DelT/(F6^2)))-1;

              end
              for i=1:size(t,1)
    if round(Quadratic_a(i)*10^34)/10^34==0
        root1(i)=-Quadratic_c(i)/Quadratic_b(i);
        root2(i)=-Quadratic_c(i)/Quadratic_b(i);
    else
       root1(i)=-(Quadratic_b(i)+sqrt((Quadratic_b(i))^2-...

(4*Quadratic_a(i)*Quadratic_c(i))))/(2*Quadratic_a(i));
```

```
        root2(i)=-(Quadratic_b(i)-sqrt((Quadratic_b(i))^2-...
(4*Quadratic_a(i)*Quadratic_c(i))))/(2*Quadratic_a(i));
    end
                Nxbt{R,i} = [root1(i);root2(i)];
                end
                if min(Nxbt{R,2})<min(Nxbt{R,3})
                    Nxbt{R,2}=Nxbt{R,3};
                elseif min(Nxbt{R,2})>min(Nxbt{R,3})
                    Nxbt{R,3}=Nxbt{R,2};
                else
                    Nxbt{R,2}=Nxbt{R,3};
                end

        for i=1:size(t,1)
        Nxbt{R,i}(Nxbt{R,i}<0)=0;
        end

        for i=1:size(t,1)
          if prod(Nxbt{R,i})>0
            Nxt(R,i)=min(Nxbt{R,i});
          else
            Nxt(R,i)=sum(Nxbt{R,i});
          end
        end
        for i=1:size(t,1)
        if R>2
            for i=1:size(t,1)
            if Nxt(R,i)==0
                Nxt(R,i)=0;
            else
                Nxt(R,i)=Nxt(R,i);
            end
            end
        else
            Nxt(R,i)=Nxt(R,i);
            Nx_rec(R,i)=Nxt(R,i)*sum(t);
        end
        end
        for i=1:size(t,1)
            if Nxt(R,i)==0
                Nxt(R,i)=20e20;
            end
        end
        Nxtn(R,:)=min(Nxt(R,:));
        Nx=sum(Nxtn(:,:))*sum(t);
```

```
        for i = 1:size(t,1)
        sigxy_1{i} =
((sigxy_FT{i})*(Nx/sum(t,1)))+(sigxy_ST{i}*(DelT));
        end

        for i = 1:size(t,1)
        sig_1{i} = (T_sig{i}*sigxy_1{i});
        end

         Ny=k1*Nx;
         Nxy=k2*Nx;
         Mx=k3*Nx;
         My=k4*Nx;
         Mxy=k5*Nx;
         N = [Nx ; Ny ; Nxy];
         M = [Mx ; My ; Mxy];
         Nbar = (N + NT + NH);
         Mbar = (M + MT + MH);
         eps0_k = (abd * [Nbar ; Mbar]);
         eps0 = (eps0_k(1:3));
         k    = (eps0_k(4:6));

        for i = 1:size(t,1)
        eps{i,1} = (eps0 + curv_mat(i)*k);
        end

        for i = 1:size(t,1)
        eps12{i,1} = T_eps{i}*eps{i,1};
        end

        for i=1:size(t,1)
            sig_term{i,1}=Q12{i}*eps12{i};
        end

        for i=1:size(t,1)
        Y_S_fiber_term(i,1) = ((sig_1{i}(1))/(F1(i)));
        Y_S_mixed_term(i,1) = (sig_1{i}(1)*sig_1{i}(2)*(B12));
        Y_S_matrix_term(i,1) = ((sig_1{i}(2))/(F2(i)));
        Y_S_shear_term(i,1) = ((sig_1{i}(3)^2)/(F6^2));
        Y_S(i,1) = Y_S_fiber_term(i,1)+Y_S_mixed_term(i,1) ...
                   +Y_S_matrix_term(i,1)+Y_S_shear_term(i,1);

        if round(Y_S(i,1)*10^1)/10^1>=1
           if Y_S_fiber_term(i,1)>Y_S_matrix_term(i,1)
                 Q12{i}(1,1)=0;
                 Q12{i}(1,2)=0;
                 Q12{i}(2,1)=0;
```

```
            Q12{i}(3,3)=0;
            Q12{i}(2,2)=(Q12{i}(2,2));
      elseif Y_S_matrix_term(i,1)>Y_S_fiber_term(i,1)
            Q12{i}(1,1)=(Q12{i}(1,1));
            Q12{i}(1,2)=0;
            Q12{i}(2,1)=0;
            Q12{i}(3,3)=0;
            Q12{i}(2,2)=0;
      end
   elseif round(Y_S(i,1)*10^1)/10^1<1
            Q12{i}(1,1)=(Q12{i}(1,1));
            Q12{i}(1,2)=(Q12{i}(1,2));
            Q12{i}(2,1)=(Q12{i}(2,1));
            Q12{i}(3,3)=(Q12{i}(3,3));
            Q12{i}(2,2)=(Q12{i}(2,2));
   end


   fiber_matrix(i,1)=E1(i,1)+E2(i,1);
   fiber_shear(i,1)=E1(i,1)+G12(i,1);
   matrix_shear(i,1)=E2(i,1)+G12(i,1);
   fiber_matrix_shear(i,1)=E1(i,1)+E2(i,1)...
       +G12(i,1);

   end
   record_of_Y_S_1(R,:)=Y_S(1,:);
   record_of_Y_S_2(R,:)=Y_S(2,:);
   record_of_Y_S_fiber_term_1(R,:)=Y_S_fiber_term(1,:);
   record_of_Y_S_matrix_term_1(R,:)=Y_S_matrix_term(1,:);

   Y_S;

eps_jones(R,:)=Nx*a(1,1)*100;


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%%% Reapplying Nx to check more laminae failures at same load
%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%

Reapply_Nx_Y_S=7;
   while Reapply_Nx_Y_S>0

   NT=zeros;
   NH=zeros;
```

```
        Ny=k1*Nx;
        Nxy=k2*Nx;
        Mx=k3*Nx;
        My=k4*Nx;
        Mxy=k5*Nx;
        N = [Nx ; Ny ; Nxy];
        M = [Mx ; My ; Mxy];

        A=0;B=0;D=0;NT=0;NH=0;MT=0;MH=0;

        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
        %%% Setting the (h) Matrix  %%%
        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

        x=zeros(size(t,1)+1,1);
        x(1,1)=sum(t)/2;
        for i=2:size(t,1)+1
            x(i)=x(i-1)-t(i-1);
        end
        h=flipud(x);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%%% Calculating (Qbar)'s, (A,B,& D) Matrices,NT, NH, MT, & MH
%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%

        k=1;
        for i=1:size(t,1)
            Alpha = [Alpha1(i);Alpha2(i);0];
            Beta =  [Beta1(i);Beta2(i);0];
            S12{i} =[1/E1(i,1)             -v12(i,1)/E1(i,1)
0; ...
                 -v12(i,1)/E1(i,1)       1/E2(i,1)         0;
...
        0                        0                  1/G12(i,1)];

       T_sig{i} =[(cos(th(i,1)))^2  (sin(th(i,1)))^2 ...
      (2*cos(th(i,1))*sin(th(i,1))); ...
  (sin(th(i,1)))^2  (cos(th(i,1)))^2 -
2*cos(th(i,1))*sin(th(i,1)); ...
      -1*cos(th(i,1))*sin(th(i,1))  1*cos(th(i,1))*sin(th(i,1))
...
      ((cos(th(i,1)))^2)-((sin(th(i,1)))^2)];
            T_sig_in{i} = inv(T_sig{i});
      T_eps{i} = [(cos(th(i,1)))^2  (sin(th(i,1)))^2 ...
```

```
     (1*cos(th(i,1))*sin(th(i,1))); ...
 (sin(th(i,1)))^2  (cos(th(i,1)))^2 -
1*cos(th(i,1))*sin(th(i,1)); ...
    -2*cos(th(i,1))*sin(th(i,1))  2*cos(th(i,1))*sin(th(i,1))
...
    ((cos(th(i,1)))^2)-((sin(th(i,1)))^2)];
            T_eps_in{i} = inv(T_eps{i});


            Qbar{i} =T_sig_in{i} * Q12{i} * T_eps{i};
            Sbar{i} = inv(Qbar{i});

            A = (A + Qbar{i} * (h(i+1,1) - h(i,1)));
            B = (B + 1/2 * Qbar{i} * (h(i+1,1)^2 - h(i,1)^2));
            D = (D + 1/3 * Qbar{i} * (h(i+1,1)^3 - h(i,1)^3));

            Alpha_xy{i} = T_sig_in{i}*Alpha;
            Beta_xy{i} = T_sig_in{i}*Beta;

            NTp(:,k) = (DelT * Qbar{i} * T_sig_in{i} * Alpha
*...
                (h(i+1,1) - h(i,1)));
            NHp(:,k) = (DelC * Qbar{i} * T_sig_in{i} * Beta
*...
                (h(i,1) - h(i+1,1)));
            MTp(:,k) = (1/2 * DelT * Qbar{i} * T_sig_in{i} *
Alpha...
                * (h(i,1)^2 - h(i+1,1)^2));
            MHp(:,k) = (1/2 * DelC * Qbar{i} * T_sig_in{i} *
Beta...
                * (h(i,1)^2 - h(i+1,1)^2));

            NT = (NT + DelT * Qbar{i} * (Alpha_xy_2eps{i})*...
                (h(i+1,1) - h(i,1)));
            NH = (NH + DelC * Qbar{i} * (Beta_xy_2eps{i}) *...
                (h(i+1,1) - h(i,1)));
            MT = (MT + 1/2 * DelT * Qbar{i} * (Alpha_xy_2eps{i})
*...
                (h(i+1,1)^2 - h(i,1)^2));
            MH = (MH + 1/2 * DelC * Qbar{i} * (Beta_xy_2eps{i})
*...
                (h(i+1,1)^2 - h(i,1)^2));

            NT_th_DelT = (NT/(sum(t)*DelT));

            k=k+1;
        end
```

```
Nbar = (N + NT + NH);
Mbar = (M + MT + MH);
ABD = ([A B;B D]);
rec_A11(R,:)=ABD(1,1);

Load_Coeff = [1;k1;k2;k3;k4;k5];
abd = pinv(ABD);
rec_a11(R,:)=abd(1,1);
a = (abd(1:3,1:3));
b = (abd(1:3,4:6));
bT= (abd(4:6,1:3));
d = (abd(4:6,4:6));
ab = ([a b]);

eps0_k = (abd * [Nbar ; Mbar]);
eps0 = (eps0_k(1:3));
k    = (eps0_k(4:6));
Alpha_bar = (1/DelT * ( a * NT + b * MT));
Beta_bar  = (1/DelC * ( a * NH + b * MH));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% Setting the (z) Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

z(1,1)=sum(t)/2;
for i=2:size(t,1)
    z(i-1,1)=z(i-1);
    z(i-1,2)=z(i-1)-t(i-1);
    z(i,1) = z(i-1,2);
end
z(size(t,1),2) = -sum(t)/2;
x=flipud(z);
x(:,[1,2])=x(:,[2,1]);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Calculating [sig]k & [eps]k for each ply %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

for i = 1:size(t,1)
eps{i,1} = (eps0 + curv_mat(i)*k);
end


for i = 1:size(t,1)
```

```
        sigxy{i} = (Qbar{i}*(eps{i,1}-(DelT*
(Alpha_xy_2eps{i}))));
        end

        for i = 1:size(t,1)
        sig{i} = (T_sig{i}*sigxy{i});
        end


        eps0_k_mech = (abd * [N ; M]);
        eps0_mech = (eps0_k(1:3));
        k_mech    = (eps0_k(4:6));

        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
        % Calculating [sig]k & [eps]k for each ply %
        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

        for i = 1:size(t,1)
        eps{i,1} = (eps0 + curv_mat(i)*k);
        end

        for i = 1:size(t,1)
        sigxy{i,1} = (Qbar{i}*eps{i,1});
        end

        for i=1:size(t,1)
            sig_a_f{R+1,i}=T_sig{i}*sigxy{i};
        end

         for i=1:size(t,1)

              if (sig{i,1}(1,1)>=0)
              F1(i)=F1t;
              else
              F1(i)=F1c;
              end
              if (sig{i,1}(2,1)>=0)
              F2(i)=F2t;
              else
              F2(i)=F2c;
              end
         end

        for i=1:size(t,1)
        Y_S_fiber_term(i,1) = ((sig{i}(1))/(F1(i)));
        Y_S_mixed_term(i,1) = (sig{i}(1)*sig{i}(2)*(B12));
        Y_S_matrix_term(i,1) = ((sig{i}(2))/(F2(i)));
```

```
        Y_S_shear_term(i,1) = ((sig{i}(3)^2)/(F6^2));
        Y_S(i,1) = Y_S_fiber_term(i,1)+Y_S_mixed_term(i,1) ...
                   +Y_S_matrix_term(i,1)+Y_S_shear_term(i,1);
        end
        for i=1:size(t,1)
        if round(Y_S(i,1)*10^1)/10^1>=1
           if Y_S_fiber_term(i,1)>Y_S_matrix_term(i,1)
                 Q12{i}(1,1)=0;
                 Q12{i}(1,2)=0;
                 Q12{i}(2,1)=0;
                 Q12{i}(3,3)=0;
                 Q12{i}(2,2)=(Q12{i}(2,2));
           elseif Y_S_matrix_term(i,1)>Y_S_fiber_term(i,1)
                 Q12{i}(1,1)=(Q12{i}(1,1));
                 Q12{i}(1,2)=0;
                 Q12{i}(2,1)=0;
                 Q12{i}(3,3)=0;
                 Q12{i}(2,2)=0;
           end
        elseif round(Y_S(i,1)*10^1)/10^1<1
                 Q12{i}(1,1)=(Q12{i}(1,1));
                 Q12{i}(1,2)=(Q12{i}(1,2));
                 Q12{i}(2,1)=(Q12{i}(2,1));
                 Q12{i}(3,3)=(Q12{i}(3,3));
                 Q12{i}(2,2)=(Q12{i}(2,2));
        end


        fiber_matrix(i,1)=E1(i,1)+E2(i,1);
        fiber_shear(i,1)=E1(i,1)+G12(i,1);
        matrix_shear(i,1)=E2(i,1)+G12(i,1);
        fiber_matrix_shear(i,1)=E1(i,1)+E2(i,1)...
            +G12(i,1);

        end
        record_of_Y_S_1(R,:)=Y_S(1,:);
        record_of_Y_S_2(R,:)=Y_S(2,:);
        record_of_Y_S_fiber_term_1(R,:)=Y_S_fiber_term(1,:);
        record_of_Y_S_matrix_term_1(R,:)=Y_S_matrix_term(1,:);

        Y_S;


for i=1:size(t,1)
    if Q12{i}(1,1)==0
        sig_a_f{R+1,i}(1,1)=0;
```

```
            sig_a_f{R+1,i}(3,1)=0;
        end
        if Q12{i}(2,2)==0
            sig_a_f{R+1,i}(2,1)=0;
            sig_a_f{R+1,i}(3,1)=0;
        end
    end


            for i=1:size(t,1)
        if round(Y_S(i,1)*10^1)/10^1>=1
            Reapply_Nx(i,1)=1;
        else
            Reapply_Nx(i,1)=0;
        end
            end
        Reapply_Nx_Y_S=sum(Reapply_Nx);
            end

        k1
        B12

    R=R+1;
        th_theta=th*180/pi();
    for i=1:size(t,1)
    if k1==0
       if th_theta(i)>=(-31) && th_theta(i)<=(31)
            if Q12{i}(1,1)<1e-15
              newvar1(i)=0;
              newvar2(i)=0;
            end

       elseif th_theta(i)>(31) && th_theta(i)<=(59)
            if Q12{i}(1,1)<1e-15
              newvar1(i)=0;
              newvar2(i)=0;
            end
            if Q12{i}(2,2)<1e-15
              newvar1(i)=0;
              newvar2(i)=0;
            end
       elseif th_theta(i)<(-31) && th_theta(i)>=(-59)
            if Q12{i}(1,1)<1e-15
              newvar1(i)=0;
              newvar2(i)=0;
            end
            if Q12{i}(2,2)<1e-15
```

```
                newvar1(i)=0;
                newvar2(i)=0;
              end
         elseif th_theta(i)<=(90) && th_theta(i)>(59)
              if Q12{i}(2,2)<1e-15
                newvar1(i)=0;
                newvar2(i)=0;
              end
         elseif th_theta(i)>=(-90) && th_theta(i)<(-59)
              if Q12{i}(2,2)<1e-15
                newvar1(i)=0;
                newvar2(i)=0;
              end
         end
      elseif k1~=0
         if th_theta(i)>=(-31) && th_theta(i)<=(31)
              if Q12{i}(1,1)<1e-15
                newvar1(i)=0;
              end
              if Q12{i}(2,2)<1e-15
                newvar2(i)=0;
              end
         elseif th_theta(i)>(31) && th_theta(i)<=(59)
              if Q12{i}(1,1)<1e-15
                newvar1(i)=0;
                newvar2(i)=0;
              end
              if Q12{i}(2,2)<1e-15
                newvar1(i)=0;
                newvar2(i)=0;
              end
         elseif th_theta(i)<(-31) && th_theta(i)>=(-59)
              if Q12{i}(1,1)<1e-15
                newvar1(i)=0;
                newvar2(i)=0;
              end
              if Q12{i}(2,2)<1e-15
                newvar1(i)=0;
                newvar2(i)=0;
              end
         elseif th_theta(i)<=(90) && th_theta(i)>(59)
              if Q12{i}(1,1)<1e-15
                newvar2(i)=0;
              end
              if Q12{i}(2,2)<1e-15
                newvar1(i)=0;
              end
```

```
    elseif th_theta(i)>=(-90) && th_theta(i)<(-59)
         if Q12{i}(1,1)<1e-15
           newvar2(i)=0;
         end
         if Q12{i}(2,2)<1e-15
           newvar1(i)=0;
         end
    end
 end
 end
     s1=sum(newvar1(:));
     s2=sum(newvar2(:));
     if s1>=1
       s1=1
     end
     if s2>=1
     s2=1
     end
     s=s1+s2;
 end

     eps_jones(R,:)=Nx*a(1,1)*100;

     if R>4
         s=0;
     end

     %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
     %          Progressive load looping         %
     %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

     N_failure_Nx(RR,:)=(sum(Nxtn));
     N_failure_Ny(RR,:)=sum(Nxtn)*k1;
     rec_k1(RR,:)=k1;
     N_knee(RR,:)=Nxtn(2,1)*sum(t,1);
     rec_A11_initial_stiffness(RR,:)=rec_A11(2,1)/sum(t);
     rec_A11_final_stiffness(RR,:)=rec_A11(size(rec_A11,1)-
 1,1)/sum(t);

     if k1<1
        k1=k1+k1_intervals;
     elseif k1==1
        k1=k1;
     else
         k1_x=k1_x-k1_intervals;
         k1=1/(k1_x);
     end
```

```
 RR=RR+1;

sig_theta=N_failure_Nx
sig_Z=N_failure_Ny
x=sig_theta;
y=sig_Z;

plot(x,y(:, B12_R), 'Color', colorstring(B12_R))
axis([0 2e9 0 2e9]);
end
```

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REFERENCES

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